Homework 3

1. **Consider the following variant of the rock game. There are two players and two piles of rocks, and each player can take a total of up to two rocks off (distributed however that player wants); thus, in a given turn a player can take 2 rocks off of one pile, 1 rock of each pile, or 1 rock off of one pile. The player to remove the last rock wins. Determine who has a winning strategy for all cases with up to 10 rocks on each pile.**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| **0** | **N/A** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** |
| **1** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** |
| **2** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** |
| **3** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** |
| **4** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** |
| **5** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** |
| **6** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** |
| **7** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** |
| **8** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** |
| **9** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** |
| **10** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** | **B** | **A** | **A** |

A = Player 1

B = Player 2

1. **Let f(n) be defined by f(1)=9 and f(n)=f(n-1)+4 if n>1. Prove that f(n)=4n+5 for all positive integers n.**

Let g(n) = 4n+5

We want to prove that f(n) = g(n) for all n > 0

Base Case: g(1) = 4\*1 + 5 = 9 = f(1)

Induction Proof: By I.H. f(n) = g(n)

f(n+1) = f(n)+4, by def.

f(n+1) = g(n)+4, by I.H.

f(n+1) = 4n+5+4, by def.

f(n+1) = 4(n+1)+5, by arith.

f(n+1) = g(n+1)

1. **Let F(a,b) be defined for positive integers a and b, by**
   * **F(1,x)=F(x,1)=x for all x**
   * **F(a,b) = F(a-1,b) +F(a,b-1) for all positive integers a and b**

**For this function F(.,.), do the following:**

* 1. **Compute F(6,6).**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** | **5** | **6** |
| **1** | 1 | 2 | 3 | 4 | 5 | 6 |
| **2** | 2 | 4 | 7 | 11 | 16 | 22 |
| **3** | 3 | 7 | 14 | 25 | 41 | 63 |
| **4** | 4 | 11 | 25 | 50 | 91 | 154 |
| **5** | 5 | 16 | 41 | 91 | 182 | 336 |
| **6** | 6 | 22 | 63 | 154 | 336 | 672 |

F(6,6) = 672

* 1. **Give a polynomial time algorithm to compute F(a,b) for arbitrary values of a and b (so the running time must be polynomial in a and b).**

Since it is constant time to look up and add two values, for any one value we have to compute it’s constant time. When we compute this over the whole table, it takes a \* b time. So, this is polynomial time with an algorithm O(a\*b).

1. **Write the following as a graph-theoretic problem: You want to divide up the class into disjoint sets so that no two people in any set are friends, and so that you use the smallest number of sets possible. (What are the vertices, what are the edges, and what are you looking for?)**

The vertices would be individual people. The edges would be relationships/friendships between people. You are then looking to split those vertices into subsets where they have no internal edges within the set. This is similar to a Bipartite graph, but with n number of subsets instead of just two.

1. **Write the following as a graph-theoretic problem: You want to find out if it is possible to divide the students in the class into disjoint pairs (everyone in some pair, but no one in two pairs) so that for each pair you create, the two students like each other. What are the vertices, what are the edges, and what are you looking for?**

The vertices would be individual people. The edges would be the “like-ness” between people, or whether or not someone liked each other. Then you are looking for subsets with exactly two vertices with an edge between the vertices within the subset.